

Holographic Cosmology from Bionic Solutions

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Abstract

In this paper, we will use a Bionic solution for analysing the holographic cosmology. A Bionic solution is a configuration of a D3-brane and an anti-D3-brane connected by a wormhole, and holographic cosmology is a recent proposal to explain cosmic expansion by using the holographic principle. In our model, a Bionic configuration will be produced by the transition of fundamental black strings. The formation of a Bionic configuration will cause inflation. As the D3-brane moves away from the anti-D3-brane, the wormhole will get annihilated, and the inflation will end with the annihilation of this wormhole. However, it is possible for a D3-brane to collide with an anti-D3-brane. Such a collision will occur if the distance between the D3-brane and the anti-D3-brane reduces, and this will create tachyonic states. We will demonstrate that these tachyonic states will lead to the formation of a new wormhole, and this will cause acceleration of the universe before such a collision.

1 Introduction

The low energy effective field theory action for D-branes is the Dirac-Born-Infeld (DBI) action [1]-[2]. This non-linear action can be used to construct a Bionic solution. A Bionic solution is a configuration of a D3-brane and an anti-D3-brane connected by a wormhole [3]-[5]. In a Bionic solution, the F-string end on a point of a D-brane, and the F-string charge gets associated with the world-volume electric flux carried by the D-brane. Thus, the F-strings (which are one-dimensional objects) become higher-dimensional brane wrapped on a sphere. This phenomena critically depends on the non-linearity of the DBI action. Such brane configurations has also been used for analysing various

aspects of the *AdS/CFT* correspondence [6]-[8], including giant gravitons [9]-[10]. This is because it is possible for gravitons (which satisfy the BPS bound) to become giant gravitons, if they are moved on the equator of the five sphere in the $AdS_5 \times S^5$ background [9]-[10]. It may be noted that Wilsons loops have also been used for analysed various D-branes configurations [11]-[12]. Thus, D-brane configurations have been used for analyzing various interesting physical systems. In this paper, we will use the Bionic solutions for analysing the holographic cosmology.

There is a close relation between geometry and thermodynamics, and this relation between the geometry and thermodynamics is the basis of the Jacobson's formalism [13]. In this formalism, gravity is described as an emergent thermodynamic force, and the Einstein equation are obtained from the Clausius relation. Thus, the structure of spacetime become an emergent structure in this formalism. It may be noted that this thermodynamic approach has led to the development of the Verlinde formalism [14]. Furthermore, the holographic cosmology is based on this formalism and the holographic principle [15]-[16]. The holographic principle states that the number of degrees of freedom in a region of space is equal to the number of degrees of freedom on the boundary surrounding that region of space. So, it is possible to use the difference between the degrees of freedom in a region of space and the degrees of freedom on the boundary surrounding that region of space to explain the cosmic acceleration, and this proposal is called the holographic cosmology [15]-[16].

The original proposal of holographic cosmology has been used in several modified theories of gravity. In fact, the Friedmann equations in Gauss-Bonnet gravity (and even more general Lovelock gravity) have been obtained using the holographic cosmology [17]. Furthermore, the brane world models [18], cosmological models in scalar-tensor gravity [19], and cosmological models in $f(R)$ gravity [20], have been studied using the Jacobson's thermodynamic approach. As the holographic cosmology is based on the Jacobson's thermodynamic approach, brane world models, cosmological model in scalar tensor gravity, and cosmological model in $f(R)$ gravity, have been analysed using the holographic cosmology [21]. A generalization of the original proposal for the holographic cosmology has been used for deriving Friedmann equation corresponding to the Friedmann-Robertson-Waker universe with an arbitrary spatial curvature [22]. The Friedmann equation for Gauss-Bonnet gravity (and even more general Lovelock gravity) with an arbitrary spatial curvature has also been derived using this generalization proposal [23]. However, it has been demonstrated that such a generalization is only valid if the aerial volume is used instead of the proper volume [24].

It has also been possible to analyse the holographic cosmology using the Bionic solution [25]. In this model, the D3-brane represent the universe, and the degrees of freedom on this brane are controlled by the evolution of Bionic solution. It may be noted that if a D3-brane is away from an anti-D3-brane, then the spike of the D3-brane are separated from the spike of the anti-D3-brane. However, as the distance between the D3-brane and the anti-D3-brane reduces to a critical value, the spike of D3-brane meets the spike of the anti-D3-brane, and a wormhole is formed [26]-[28]. This configuration of a D3-brane and an anti-D3-brane joined by a wormhole is called a BIon. It is possible for this wormhole to act as a channel for the degrees of freedom to flow into the D3-brane, and this can cause the cosmic acceleration. So, we will analyse the

holographic cosmology using a Bionic solution [26]-[27]. In fact, we will use a thermal generalization of the usual Bionic solution for analyzing holographic cosmology. This is because the BIon is a static solution, but we need a dynamic parameter in this solution to relate it to the time evolution of our universe. As the temperature of the universe decreases during its evolution, we can identify the cosmological clock with the temperature of the Bionic solution. In our model, the BIon will first form from fundamental black strings. The D3-brane in this BIon will represent our universe. The inflation will occur because the degrees of freedom will flow into the the D3-brane. As the D3-branes moves away from the anti-D3-brane, the spike of D3-brane will get separating from the spike of anti-D3-brane, and this will annihilate the wormhole. The inflation will end when the wormhole gets annihilated. However, it is possible for a D3-brane to collide with an anti-D3-brane. We will demonstrate that tachyonic states will form as a D3-brane approaches an anti-D3-brane, and the existence of these tachyonic states will lead to the formation of a new wormhole. This will cause the late time acceleration of the universe before such an collision.

The the paper is organized as the follows. In section 2, we discuss the emergence of BIon from fundamental black strings, and use it for analyzing holographic inflationary cosmology. In section 3, we analyse the tachyonic states in theory and analyse their consequences. In the last section, we summarize our results and discuss some possible extensions of the results obtained in this paper.

2 Holographic Bionic Solutions

In this section, we will analyse the holographic cosmology using Bionic solutions. We will start from k fundamental black strings, and a BIon will be formed from a transition of these k fundamental black strings. The supergravity solution for k coincident non-extremal black F-strings (lying along the z direction) can be written

$$\begin{aligned} ds^2 &= H^{-1}(-f dt^2 + dz^2) + f^{-1} dr^2 + r^2 d\Omega_7^2, \\ e^{2\phi} &= H^{-1}, \quad B_0 = H^{-1} - 1, \\ H &= 1 + \frac{r_0^6 \sinh^2 \alpha}{r^6}, \\ f &= 1 - \frac{r_0^6}{r^6}. \end{aligned} \quad (1)$$

The mass density (along the z direction) for this solution is given by [27],

$$\frac{dM_{F1}}{dz} = T_{F1} k + \frac{16(T_{F1} k \pi)^{3/2} T^3}{81 T_{D3}} + \frac{40 T_{F1}^2 k^2 \pi^3 T^6}{729 T_{D3}^2}. \quad (2)$$

It may be noted that this equation is valid only up to order T^6 (using small temperature limit). We will use this approximation through out this paper, and higher order corrections will produce corrections terms to the expressions obtained in this paper. However, they will not change any quantitative feature of the our model. The metric in which the BIon is embedded can be written as [26],

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \sum_{i=1}^6 dx_i^2. \quad (3)$$

Choosing the worldvolume coordinates of the D3-brane as $\{\sigma^a, a = 0..3\}$ and defining $\tau = \sigma^0$, $\sigma = \sigma^1$, the coordinates of BIon are given by [26],

$$t(\sigma^a) = \tau, r(\sigma^a) = \sigma, x_1(\sigma^a) = z(\sigma), \theta(\sigma^a) = \sigma^2, \phi(\sigma^a) = \sigma^3. \quad (4)$$

The remaining coordinates $x_{i=2,..6}$ are constant in our model. The embedding function $z(\sigma)$ describes the bending of the brane. The induced metric on the brane can then be written as

$$\gamma_{ab}d\sigma^a d\sigma^b = -d\tau^2 + (1 + z'(\sigma)^2)d\sigma^2 + \sigma^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

where z is a transverse coordinate to the branes and σ is its radius. So, the spatial volume element can be expressed as $dV_3 = \sqrt{1 + z'(\sigma)^2}\sigma^2 d\Omega_2$. Now we impose the boundary conditions, $z(\sigma) \rightarrow 0$ for $\sigma \rightarrow \infty$ and $z'(\sigma) \rightarrow -\infty$ for $\sigma \rightarrow \sigma_0$, where σ_0 is the minimal two-sphere radius. The mass density of the BIon along the z direction is given by

$$\frac{dM_{BIon}}{dz} = T_{F1}k + \frac{3\pi T_{F1}^2 k^2 T^4}{32T_{D3}^2 \sigma_0^2} + \frac{7\pi^2 T_{F1}^3 k^3 T^8}{512T_{D3}^4 \sigma_0^4}. \quad (6)$$

This equation is valid only up to order T^6 , and thus all the equations obtained using this equation are also valid to the order T^6 . It may be noted that the mass density of the BIon given is equal to the mass density of the F-strings at $\sigma = \sigma_0$. This is because the thermal BIon is formed from k F-strings at $\sigma = \sigma_0$. Thus, at this point we can identify the thermodynamics of the BIon with the thermodynamics of k F-strings. To match two Eq. (2) with Eq. (6) at this point, σ_0 should have the following temperature dependence

$$\sigma_0 = \left(\frac{\sqrt{kT_{F1}}}{T_{D3}} \right)^{1/2} \sqrt{T} \left[C_0 + C_1 \frac{\sqrt{kT_{F1}}}{T_{D3}} T^3 \right], \quad (7)$$

where $T_{F1} = 4k\pi^2 T_{D3} g_s l_s^2$, and C_0, C_1, F_0, F_1, F_2 are numerical coefficients which can be obtained from T^3 and T^6 terms in Eqs. (2) and (6). It may be noted that from Eq. (7), σ is a function of the temperature, and so the metric is a dynamic function of the temperature. Furthermore, σ can also be related to the width of the wormhole, and so at a critical temperature T_{end} , the wormhole gets annihilated

$$\sigma_0 = 0, \bar{C}_0 = -C_0 \rightarrow T_{end} = \frac{\bar{C}_0 \sqrt{T_{D3}}}{C_1 k T_{F1}}. \quad (8)$$

It may be noted that we have identified the temperature of the BIon solution with the cosmological clock. This is because the temperature of the BIon solution is very high at the beginning of inflation, and then reduces with the evolution of the universe. Hence, a cosmological clock can be identified with the temperature of the BIon solution [26]-[28]. In fact, the temperature is infinite at the beginning, and it decreases to T_{end} at the end of inflation. The width of the wormhole also decreases with time, and at $T = T_{end}$ the wormhole is annihilated. The annihilation of the wormhole occurs due to the D3-brane moving away from the anti-D3-brane. This is because when the D3-brane is close to the anti-D3-brane there spikes meet, forming a wormhole. However, as

the D3-brane moves away from the anti-D3-brane, there spikes move away from each other. So, at a critical point the wormhole gets annihilated.

The inflation occurs because the wormhole acted as a channel for the degrees of freedom to flow into the D3-brane. Putting k F-string charges along the radial direction and using Eq. (7), we obtain [27],

$$z(\sigma) = \int_{\sigma}^{\infty} d\sigma \left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}}. \quad (9)$$

Now for a thermal BIon, $F(\sigma)$ can be expressed as

$$F(\sigma) = \sigma^2 \frac{4 \cosh^2 \alpha - 3}{\cosh^4 \alpha}, \quad (10)$$

where $\cosh \alpha$ is given by

$$\cosh^2 \alpha = \frac{3 \cos \frac{\delta}{3} + \sqrt{3} \sin \frac{\delta}{3}}{2 \cos \delta}, \quad (11)$$

Here we have defined $\cos \delta$ as

$$\begin{aligned} \cos \delta &\equiv \overline{T}^4 \sqrt{1 + \frac{k^2}{\sigma^4}}, & \overline{T} &\equiv \left(\frac{9\pi^2 N}{4\sqrt{3}T_{D_3}} \right) T, \\ k &\equiv \frac{kT_{F1}}{4\pi T_{D_3}}. \end{aligned} \quad (12)$$

where T is the temperature of BIon, N is the number of D3-branes, T_{D_3} is the tension of the brane, and T_{F1} is tensions of the fundamental string. It is possible to attach a mirror solution to Eq. (9), and obtain a wormhole configuration. Now $\bar{\Delta} = 2z(\sigma_0)$, is the distance separating N D3-branes from N anti-D3-branes for a given BIon configuration. This configuration is thus defined by four parameters N , k , T and σ_0 ,

$$\bar{\Delta} = 2z(\sigma_0) = 2 \int_{\sigma_0}^{\infty} d\sigma \left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}}. \quad (13)$$

In the small temperature limit, we obtain

$$\bar{\Delta} = \frac{2\sqrt{\pi}\Gamma(5/4)}{\Gamma(3/4)}\sigma_0 \left(1 + \frac{8}{27} \frac{k^2}{\sigma_0^4} \overline{T}^8 \right). \quad (14)$$

Now we can construct a holographic cosmological model using this BIon solution [15, 16, 25]. The total entropy during cosmic expansion can be obtained by adding the number of degrees of freedom in the bulk with the number of degrees of freedom on the boundary. It may be noted that by surface degrees of freedom we mean the degrees of freedom on the apparent horizon of the universe. Furthermore, according to the holographic cosmology, the difference between the number of degrees of freedom in the bulk and the number of degrees of freedom on the boundary is equal to the mass density [15, 16, 25]. We can obtain the mass density by placing the wormhole along the z -axis [15, 16, 25]. In this paper, this sum is equated with the total degrees of freedom of the BIon,

as we are using a BIon to model the inflation. Thus, we can write the following equations,

$$\begin{aligned}
N_{sur} + N_{bulk} &= N_{BIon} = N_{brane} + N_{anti-brane} + N_{wormhole} \\
&\simeq 4l_P^2 S_{BIon} \\
&= \frac{4T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} \\
N_{sur} - N_{bulk} &\simeq \int d\sigma \frac{dM_{BIon}}{dz} \\
&= \frac{2T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha}.
\end{aligned} \tag{15}$$

According to this model, our universe represented by a D3-brane interacts with an anti-D3-brane through a wormhole. Thus, the total number of degrees of freedom is equal to sum of the number of degrees of freedom of the D3-brane, the anti-D3-brane, and the wormhole, $N_{total} = N_{sur,universe} + N_{sur,anti-universe} + N_{wormhole}$. Here the anti-D3-brane represents the anti-universe. So, we can define $N_{bulk} = N_{sur,anti-universe} + N_{wormhole}$. Now as our universe is located on a brane and another anti-universe is located on an anti-brane, we can write $N_{total} = N_{sur,universe} + N_{bulk} = N_{sur,universe} + N_{sur,anti-universe} + N_{wormhole} = N_{brane} + N_{anti-brane} + N_{wormhole}$. Solving these equations simultaneously, we obtain,

$$\begin{aligned}
N_{sur} &\simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} \\
&\quad + \frac{2T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha}, \\
N_{bulk} &\simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} \\
&\quad - \frac{2T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha}.
\end{aligned} \tag{16}$$

We can obtain cosmological parameters like Hubble parameter and energy density of the universe from this Blonic solution. According to the usual holographic cosmology, the number of degrees of freedom on the spherical surface of apparent horizon with radius r_A is proportional to its area [22],

$$N_{sur} = \frac{4\pi r_A^2}{l_P^2}, \tag{17}$$

where $r_A = 1/\sqrt{H^2 + \frac{\bar{k}}{a^2}}$ is the radius of the apparent horizon for the Friedmann-Robertson-Walker universe, and $H = \frac{\dot{a}}{a}$ is the Hubble parameter (a is the scale factor). So, we obtain an expression for the Hubble parameter,

$$H_{flat,inf} \simeq \frac{18\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^8} T^{12} + \frac{8\pi k^2 N^{10} T_{F1}^{12}}{T_{D3}^{12} \sigma_0^4} T^{10}. \tag{18}$$

The temperature of the universe is very high at its beginning, and the rate of expansion is also very large. This can be seen from the fact that at this stage

there is a large Hubble parameter. This is because the wormhole acts as a channel for the degrees of freedom to flow into the universe. However, at a critical temperature, the inflation ends because the wormhole get annihilated at that critical temperature.

We can calculate the energy density of the universe using the Friedmann equation for the flat Friedmann-Robertson-Waker universe,

$$\rho_{flat,inf} = \frac{3}{8\pi l_P^2} H_{flat}^2 \simeq \frac{27\pi k^8 N^{24} T_{F1}^{28}}{4l_P^2 T_{D3}^{28} \sigma_0^{16}} T^{24} + \frac{3\pi k^4 N^{20} T_{F1}^{24}}{l_P^2 T_{D3}^{24} \sigma_0^8} T^{20}. \quad (19)$$

Thus, the energy density depends on the temperature of the Blon. So, the energy density of the universe reduces as the temperature of the Blon reduces to $T = T_{end}$, at the end of the inflation. Now using (17), and assuming $r_A = 1/\sqrt{H^2 + \frac{\bar{k}}{a^2}}$, we can obtain Hubble parameter for non-flat universe in terms of temperature,

$$H_{o/c,inf} \simeq \sqrt{\left(\frac{18\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^8} T^{12} + \frac{8\pi k^2 N^{10} T_{F1}^{12}}{T_{D3}^{12} \sigma_0^4} T^{10} \right)^2 - \bar{k}/a^2}. \quad (20)$$

Furthermore, we define \mathcal{T} as

$$\mathcal{T} = \left(\frac{18\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^8} T^{12} + \frac{8\pi k^2 N^{10} T_{F1}^{12}}{T_{D3}^{12} \sigma_0^4} T^{10} \right)^2, \quad (21)$$

and solve this equation. Now we can write the scale factor for open ($k = -1$) universe as

$$a_{o,inf}(t) \simeq \exp - \int dt [\mathcal{T} + \ln(t)], \quad (22)$$

and the scale factor for closed ($k = +1$) universe as

$$a_{c,inf}(t) \simeq \exp -i \int dt [\mathcal{T} + \ln(t) + \frac{\pi}{2}]. \quad (23)$$

Thus, the behavior of open and closed universes also depends on the temperature. The scale factor of open universe is almost zero at the beginning ($T = \infty$), and it increases with decreasing temperature. At the end of the inflation, it has a larger value. However, the scale factor of closed universe oscillates during inflation. The energy density for open and closed universes during this inflation can be written as

$$\begin{aligned} \rho_{o/c,inf} &= \frac{3}{8\pi l_P^2} (H_{o/c}^2 + k/a^2) \\ &\simeq \frac{3}{8\pi l_P^2} H_{flat}^2 \\ &\simeq \frac{3}{8\pi l_P^2} \left(\frac{27\pi k^8 N^{24} T_{F1}^{28}}{4l_P^2 T_{D3}^{28} \sigma_0^{16}} T^{24} + \frac{3\pi k^4 N^{20} T_{F1}^{24}}{l_P^2 T_{D3}^{24} \sigma_0^8} T^{20} \right) \\ &= \rho_{flat,inf}. \end{aligned} \quad (24)$$

We now observe that in this model the energy density is the same for flat, open and closed universes. This is because that the energy density originates due to the evolution of a Bionic solution, and so it does not depend on type of universe.

It may be noted that when the inflation ends, the wormhole is annihilated, and the mass distribution along z -direction is absent at this stage. Thus, at this stage the surface degrees of freedom on the boundary and the bulk degrees of freedom in the universe can be expressed as

$$N_{sur} - N_{bulk} \simeq \int_{\sigma_0}^{\sigma_0} d\sigma \frac{dM_{BIon}}{dz} = 0. \quad (25)$$

So, we can write

$$N_{sur} = N_{bulk}. \quad (26)$$

Thus, the degrees of freedom on the boundary is equal to the degrees of freedom in bulk, at the end of the inflation. In fact, the inflation was caused because of the difference between the degrees of freedom on the boundary and in the bulk, and so the inflation ends when the bulk degrees of freedom equals the boundary degrees of freedom.

3 Tachyonic States

In this section, we will analyse the effect of tachyonic states in this theory. It is possible for a D3-brane to collide with an anti-D3-brane. This will occur when the distance between a D3-brane and an anti-D3-brane decreases. However, as the distance between a D3-brane and an anti-D3-brane reduces, tachyonic states will be created. These tachyonic states will form will tachyonic spikes, and when the tachyonic spikes of the D3-brane meet the tachyonic spikes of the anti-D3-brane, a tachyonic wormhole will be formed [28]. So, this new Bionic solution will consist of the tachyonic wormhole, the D3-brane and the anti-D3-brane. The wormhole in this new Bionic solution will again act as a channel for the degrees of freedom to flow into the universe. This will cause the number of degrees of freedom in the universe to increase, and that will in turn cause late time acceleration of the universe. Thus, the universe will evolved from non-phantom phase to phantom one. Now we consider a set of D3-branes and anti-D3-branes (3), which are placed at points $z_1 = l/2$ and $z_2 = -l/2$, respectively. For this system, we can write

$$\begin{aligned} S &= -\tau_3 \int d^9\sigma \sum_{i=1}^2 V(TA, l) e^{-\phi(\sqrt{-\det A_i})}, \\ (A_i)_{ab} &= (g_{MN} - \frac{TA^2 l^2}{Q} g_{Mz} g_{zN}) \partial_a x_i^M \partial_b x_i^M \\ &\quad + F_{ab}^i + \frac{1}{2Q} ((D_a TA)(D_b TA)^* + (D_a TA)^*(D_b TA)) \\ &\quad + il(g_{az} + \partial_a z_i g_{zz})(TA(D_b TA)^* \\ &\quad - TA^*(D_b TA)) + il(TA(D_a TA)^* - TA^*(D_a TA)) \\ &\quad \times (g_{bz} + \partial_b z_i g_{zz}) \left(1 - \frac{\pi^2 N T^4}{6 T_{D3}}\right). \end{aligned} \quad (27)$$

where

$$Q = 1 + TA^2 l^2 g_{zz},$$

$$D_a T A = \partial_a T A - i(A_{2,a} - A_{1,a}) T A, V(T A, l) = g_s V(T A) \sqrt{Q},$$

$$e^\phi = g_s \left(1 + \frac{R^4}{z^4}\right)^{-\frac{1}{2}}. \quad (28)$$

Here ϕ is the dilaton field, $A_{2,a}$ is the gauge field, and F_{ab}^i is the field strengths on the world-volume of the non-BPS brane. Furthermore, $T A$ is the tachyon field, τ_3 is the brane tension and $V(T A)$ is the tachyon potential. The indices a, b denote the tangent directions of D3-branes, while the indices M, N run over the background ten-dimensional space-time directions. The D3-brane and the anti-D3-brane are labeled by $i = 1, 2$, respectively. The separation between the D3-brane and the anti-D3-brane is denoted by $z_2 - z_1 = l$. Here we have chosen, $2\pi\alpha' = 1$. Now we can write a potential for this system as [29]-[34],

$$V(T A) = \frac{\tau_3}{\cosh \sqrt{\pi} T A}. \quad (29)$$

Let us only consider the σ dependence of the tachyon field $T A$, and for simplicity, we neglect the contributions coming from the gauge fields. So, the Lagrangian given by Eq. (33), in the region $r > R$ and $T A' \sim \text{constant}$, simplifies to

$$L \simeq -\frac{\tau_3}{g_s} \int d\sigma \sigma^2 V(T A) (\sqrt{D_{1,T A}} + \sqrt{D_{2,T A}}) \left(1 - \frac{\pi^2 N T^4}{6 T_{D3}}\right), \quad (30)$$

where

$$D_{1,T A} = D_{2,T A} \equiv D_{T A} = 1 + \frac{l'(\sigma)^2}{4} + \dot{T A}^2 - T A'^2. \quad (31)$$

Here we have assume that $T A l \ll T A'$. Now we will obtain the Hamiltonian corresponding to this Lagrangian, and used it for analysing this system. The canonical momentum density $\Pi = \frac{\partial L}{\partial \dot{T A}}$ associated with the tachyon is given by

$$\Pi = \frac{V(T A) \dot{T A}}{\sqrt{1 + \frac{l'(\sigma)^2}{4} + \dot{T A}^2 - T A'^2}} \left(1 - \frac{\pi^2 N T^4}{6 T_{D3}}\right). \quad (32)$$

So, the Hamiltonian can be written as

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 \Pi \dot{T A} - L.. \quad (33)$$

Thus, by choosing $\dot{T A} = 2 T A'$, we obtain

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 [\Pi (\dot{T A} - \frac{1}{2} T A')] + \frac{1}{2} T A \partial_\sigma (\Pi \sigma^2) - L. \quad (34)$$

Here we have integrated by parts the term proportional to $\dot{T A}$. So, tachyon can be studied as a Lagrange multiplier by imposing the constraint $\partial_\sigma (\Pi \sigma^2 V(T A)) = 0$ on the canonical momentum. Solving this equation, we obtain

$$\Pi = \frac{\beta}{4\pi \sigma^2}, \quad (35)$$

where β is a constant. Now substituting Eq. (41) in Eq. (40), we obtain

$$\begin{aligned} H_{DBI} &= \int d\sigma V(TA) \left(\sqrt{1 + \frac{l'(\sigma)^2}{4}} + TA^2 - TA'^2 \right) F_{DBI}, \\ F_{DBI} &= \sigma^2 \sqrt{1 + \frac{\beta}{\sigma^2}} \left(1 - \frac{\pi^2 NT^4}{6T_{D3}} \right). \end{aligned} \quad (36)$$

The resulting equation of motion for $l(\sigma)$ is given by

$$\left(\frac{l' F_{DBI}}{4 \sqrt{1 + \frac{l'(\sigma)^2}{4}}} \right)' = 0. \quad (37)$$

Solving this equation, we obtain

$$l(\sigma) = 2 \left(\frac{l_0}{2} - \int_{\sigma}^{\infty} d\sigma \left(\frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} - 1 \right)^{-\frac{1}{2}} \right). \quad (38)$$

This solution, for a non-zero σ_0 represents a wormhole with a finite size. Thus, a new wormhole is formed when the distance separating the D3-brane from the anti-D3-brane is l_0 . The width of the wormhole at this point is σ_0 . Now we can write

$$\left(\frac{1}{\sqrt{D_{TA}}} TA'(\sigma) \right)' = \frac{1}{\sqrt{D_{TA}}} \left[\frac{(V(TA)F_{DBI})}{F_{DBI}V(TA)'} (D_{TA} - TA'(\sigma)^2) \right]. \quad (39)$$

Solving this equation, we obtain

$$TA \sim \sqrt{\frac{\sigma_0^2}{\sigma_0^2 - \sigma^2}} \left(\frac{1}{1 + \frac{\pi^2 NT^4}{6T_{D3}}} \right). \quad (40)$$

So, the tachyonic states cause the formation of the wormhole at $\sigma_0 = 0$. The width of the wormhole increases as the distance between the D3-brane and the anti-D3-brane reduces, and finally the D3-brane collides with the anti-D3-brane.

Now using the action given by Eq. (36), we can obtain entropy and mass density along z -direction for this tachyonic system,

$$\begin{aligned} S_{tb} &= \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \sigma^3 \\ &\times \frac{4}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}} \end{aligned} \quad (41)$$

$$\frac{dM_{tb}}{dz} = \frac{2T_{D3}^2}{\pi T^4} V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \sigma^3 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}}. \quad (42)$$

We can analyse the effect of tachyonic potential on the number of degrees of freedom of the universe. This can be done by repeating the analysis of the previous section. Thus, we write again relate these degrees of freedom to the entropy of BIon. We also write an expression for the the mass density along the

transverse direction,

$$\begin{aligned}
N_{sur} + N_{bulk} &= N_{BIon} = N_{brane} + N_{anti-brane} + N_{wormhole} \\
&\simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \\
&\quad \times \sigma^3 \frac{4}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}} \\
N_{sur} - N_{bulk} &\simeq \int d\sigma \frac{dM_{BIon}}{dz} \\
&= \frac{2T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \sigma^3 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha} \\
&\quad \times \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}}. \tag{43}
\end{aligned}$$

Solving these equations simultaneously, we obtain,

$$\begin{aligned}
N_{sur} &\simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \\
&\quad \times \sigma^3 \frac{4}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}} \\
&\quad + \frac{2T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \\
&\quad \times \sigma^3 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}}, \\
N_{bulk} &\simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \\
&\quad \times \sigma^3 \frac{4}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}} \\
&\quad - \frac{2T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \\
&\quad \times \sigma^3 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma^2 - \sigma_0^2)^{3/2}}. \tag{44}
\end{aligned}$$

This equation demonstrates that as the D3-brane approaches the anti-D3-brane, tachyonic potential increases and tends to infinity. Thus the number of degrees of freedom becomes large as one trends to the Big Rip singularity. Thus, for this system, the Hubble parameter for flat universe can be written as

$$H_{flat,ac} \simeq \left(\frac{1}{V(TA)} \right)^{1/2} \left(\frac{72\pi k^8 N^{14} T_{F1}^{16}}{T_{D3}^{16} \sigma_0^{10}} T^{14} + \frac{8\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^6} T^{12} \right). \tag{45}$$

It may be noted that the Hubble parameter depends on tachyonic potential between the D3-brane and the anti-D3-brane. As the tachyonic potential increases, Hubble parameter reduces to a very small value.

Finally, using the Friedmann equation for the flat Friedmann-Robertson-Waker universe, we can write the energy density for this system as

$$\rho_{flat,ac} = \frac{3}{8\pi l_P^2} H^2$$

$$\simeq \frac{3}{8\pi l_P^2} \left(\frac{1}{V(TA)} \right) \left(\frac{4900\pi k^{16} N^{24} T_{F1}^{32}}{T_{D3}^{32} \sigma_0^{120}} T^{28} + \frac{16\pi k^8 N^{24} T_{F1}^{28}}{T_{D3}^{28} \sigma_0^{12}} T^{24} + \frac{496\pi k^{12} N^{26} T_{F1}^{30}}{T_{D3}^{30} \sigma_0^{16}} T^{26} \right). \quad (46)$$

The energy density decreases with increasing tachyonic states, and it reduces to zero at $TA = \infty$. This is because that D3-brane moves towards the anti-D3-brane, and this creates tachyon states, which in turn increase the radius of universe. This leads to acceleration of the universe, and it also decreases the energy density of the universe.

The properties of open and closed universes can now be obtained from Eq. (17), and $r_A = 1/\sqrt{H^2 + \frac{\bar{k}}{a^2}}$. So, the Hubble parameter for non-flat universe during this stage of evolution can be expressed as

$$H_{o/c,ac} \simeq \sqrt{\left(\frac{1}{V(TA)} \right)} \times \sqrt{\left(\frac{72\pi k^8 N^{14} T_{F1}^{16}}{T_{D3}^{16} \sigma_0^{10}} T^{14} + \frac{8\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^6} T^{12} \right)^2 - \bar{k}/a^2}. \quad (47)$$

Now we can define \mathcal{T}_t as

$$\mathcal{T}_t = \left(\frac{72\pi k^8 N^{14} T_{F1}^{16}}{T_{D3}^{16} \sigma_0^{10}} T^{14} + \frac{8\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^6} T^{12} \right)^2. \quad (48)$$

Using this definition of \mathcal{T}_t , we can write the scale factor for open universe as

$$a_{o,ac}(t) \simeq \exp - \int dt [\mathcal{T}_t + \ln(t)], \quad (49)$$

and the scale factor for closed universe as

$$a_{c,ac}(t) \simeq \exp -i \int dt [\mathcal{T}_t + \ln(t) + \frac{\pi}{2}]. \quad (50)$$

These scale factors depend on both the tachyonic potential and the temperature. Thus, by increasing tachyons, the scale factor of open universe increases and tends to infinity at $TA = \infty$, however, the scale factor of closed universe oscillates at this stage.

The energy density of the open and closed universes can be written as

$$\begin{aligned} \rho_{o/c,ac} &= \frac{3}{8\pi l_P^2} (H_{o/c,ac}^2 + k/a^2) \\ &\simeq \frac{3}{8\pi l_P^2} H_{flat,ac}^2 \\ &\simeq \frac{3}{8\pi l_P^2} \left(\frac{1}{V(TA)} \right) \left(\frac{72\pi k^8 N^{14} T_{F1}^{16}}{T_{D3}^{16} \sigma_0^{10}} T^{14} + \frac{8\pi k^4 N^{12} T_{F1}^{14}}{T_{D3}^{14} \sigma_0^6} T^{12} \right)^2 \\ &= \rho_{flat,ac}. \end{aligned} \quad (51)$$

It may be noted that again the energy density for the open universe is equal to the energy density of the closed universe. This is because this energy density depends on the tachyonic potential, and not on the specific type of the universe. Thus, we have been able to analyse the state of the universe just before a D3-brane collides with an anti-D3-brane.

4 Conclusion

In this paper, we used a configuration of a D3-brane and an anti-D3-brane connected by a wormhole for analysing the holographic inflationary cosmology. This BIon solution was obtained from fundamental black strings. The flow of degrees of freedom into the D3-brane caused inflation. This flow occurred due to the wormhole connecting the D3-brane with the anti-D3-brane, and so the inflation ended when this wormhole was annihilated. We also pointed out that it is possible for a D3-brane to collide with an anti-D3-brane. Such a collision will occur when the distance between a D3-brane and an anti-D3-brane is reduced below a critical value. However, the reduction of the distance between a D3-brane and an anti-D3-brane will lead to the formalism of tachyonic states. A new wormhole will form due to the presence of these tachyonic states, and this will cause the late time acceleration of the universe, before such a collision. It may be noted that the holographic cosmology [15]-[16] has already been used for analyzing brane world models [21], we have generalized such an analysis to a BIon solution. Even though it had been proposed that holographic cosmology can be analysed using the BIon [25], in this paper, we have analysed the detailed consequences of such a model. We have obtained explicit expression for the Hubble parameter and energy density of the universe for such a model.

It may be noted that the finite temperature effects for non-extremal self-dual string solutions and wormhole solutions interpolating between stacks of M5-branes and anti-M5-branes have also been studied, and such solutions define a BIon solution in M-theory [35]-[36]. This analysis has been done using the blackfold approach, and the self-dual string solitonic solutions have been analysed as a three-funnel solution of an effective five-brane theory. It would be interesting to study holographic cosmology using this BIon solution. Furthermore, solution to the non-abelian theory of coincident D-strings has been analysed using noncommutative geometry [37]. This was done by using funnels to the non-abelian D-string expanding out into an orthogonal D3-brane. It was also demonstrated that this configurations is dual to the BIon solutions in the abelian worldvolume theory of the D3-brane. In this paper, we have analysed the holographic cosmology using the BIon solutions, so it would be interesting to analyse the holographic cosmology using this theory which is dual to a BIon solution. It is also possible to include the effects of a nonzero $NS-NS$ two-form field in this dual theory [38]. A tilted BIon is obtained by the inclusion of such a field, and the core of this tilted BIon expands out to a single D3-brane (at an angle to the D1-brane core). It has been demonstrated that the properties of this system are consistent with that of an abelian D3-brane in a background worldvolume magnetic field. It would be interesting to analyse the holographic cosmology using such a system.

The strings have an extended structure, and the extended structure of strings is expected to act as a minimum measurable length scale in spacetime [39]. This is because the smallest probe available in string theory is the fundamental string. So, it is not possible to probe spacetime below string length scale. However, the existence of a minimum length scale in spacetime leads to the generalized uncertainty principle [40]-[48]. In fact, it has been demonstrated that the generalized uncertainty principle can be used to analyse the corrections to the AdS/CFT correspondence beyond the supergravity approximation [49]. It may be noted that the corrections to Friedmann equations coming from the generalized uncer-

tainty principle has been obtained [50]-[51]. It would be interesting to analyse the effect of extended structure of strings on the holographic cosmology. It is expected that it will deform the Bionic holographic cosmology. The effect of generalized uncertainty principle on the usual holographic cosmology has already been analysed [52]. Thus, it would be interesting to repeat that analyses for the Bionic holographic cosmology. It may be noted that even though the holographic cosmology has been studied for brane world models [18], cosmological models in scalar-tensor gravity [19], and cosmological models in $f(R)$ gravity [20], the deformation of these systems by generalized uncertainty has not been analysed. However, it is possible to study such a generalization, and it would be interesting to analyse the deformation of these cosmological models using the generalized uncertainty principle. It may be noted that cosmological and astrophysical consequences of extended theories of gravity have been studied in both metric and palatini formalism [53]. It would be interesting to relate the work done in this paper to similar cosmological and astrophysical observations. It may be noted that Λ CDM model and other observational tests have been used to constrain the current cosmic acceleration [54]. It would be important to use the Λ CDM and these other observational tests to constrain the parameters of holographic cosmology.

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